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DISTRIBUTION OF FINELY DISPERSED PARTICLES WITH RESPECT TO RESIDENCE TIME IN A VORTEX CHAMBER

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The motion of finely dispersed particles is described statistically with the use of the Fokker-Planck equation. An expression is obtained for the particle distribution function with respect to residence time. Results of the calculation illustrate the dependence of the average particle residence time in the apparatus on the process parameters.

Use of vortex flows is one of the most effective methods for intensification of interphase heat transfer in apparatus with heterogeneous systems. Vortex drying chambers provide relatively active hydrodynamic regimes with sufficiently high relative velocities of the gas and particles, developed phase contact surfaces, and a high holding capacity and residence time in comparison with cyclone apparatus and pneumatic pipes [1, 2]. Development of the notions of motion and interaction of phases in vortex drying chambers and their complexity require using various kinds of methods for their investigation. In the present work the authors describe statistical simulation of motion of the disperse phase in a vortex disk chamber with the use of random Markovian processes [3, 4]. This approach is used for investigation of the process of separation in hydrocyclones [4-6] and motion of particles in a vortex chamber [7].

We will consider the case of motion of finely disperse particles for which (Re << 1) the inertial force and the component of the resistance force caused by unsteadiness of the motion can be neglected. Thus, the resistance force can be determined by the Stokes formula. The radial motion of the particles is defined by the centrifugal force, the resistance force, and the action of random forces caused by the stochastic nature of the motion and interaction of phases (Fig. 1). With the assumptions adopted, the equation of radial motion of a particle has the form

$$\frac{dr}{d\tau} = V_r(r) + \frac{d_p^2 [\rho_p(\tau) - \rho_g] W_\varphi^2(r)}{18\mu r} + \frac{1}{3\pi\mu d_p} \xi(\tau) . \tag{1}$$

As can be seen from Eq. (1), the velocity of radial motion of the particle is the sum of a certain average value (the first two terms) and the fluctuation component. In this case the particle motion is a random process. Consequently, the particle distribution in the radial direction can be described by the distribution function $f(r, \tau)$. The quantity $f(r, \tau)dr$ determines the probability of resistance of the particle at the time τ in the annulus $[r, r+dr]$.

It will be assumed that $\xi(\tau)$ is the delta correlation function of time with zero average value and intensity C'. In this case the random process considered can be assumed to be a Markovian process [3, 4]. Then, the probability density can be determined from the Fokker-Planck equation, which with Eq. (1) is written as

$$\frac{\partial f}{\partial \tau} = - \frac{\partial}{\partial r} \left[\left(V_r + \frac{d_p^2 (\rho_p(\tau) - \rho_g) W_\varphi^2}{18\mu r} \right) f \right] + \frac{C}{2} \frac{\partial^2 f}{\partial r^2} , \tag{2}$$

where

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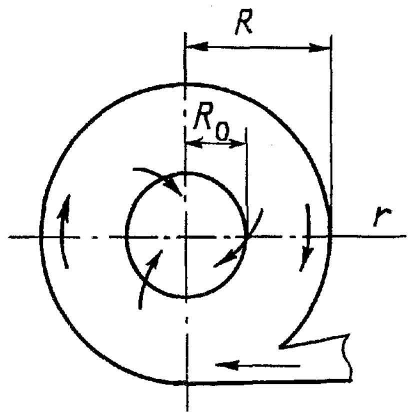


Fig. 1. Schematic diagram of a vortex chamber.

$$C = \frac{C'}{9\pi^2 \mu^2 d_p^2}.$$

The particle density is expressed in terms of the moisture content as

$$\rho_p(\tau) = \rho_0 \frac{1 + u(\tau)}{1 + u_{in}}. \tag{3}$$

The moisture content of the particle is determined by the formula [8]

$$\frac{du}{d\tau} = -K(u - u_{eq}).$$

Integration gives

$$u(\tau) = u_{eq} + (u_{in} - u_{eq}) e^{K\tau}. \tag{4}$$

Solution of Eq. (2) should satisfy the boundary conditions

$$f = 0 \quad \text{at} \quad r = R_0, \tag{5}$$

$$\left(V_r + \frac{d_p^2 (\rho_p(\tau) - \rho_g) W_\varphi^2}{18 \mu r} \right) f - \frac{C}{2} \frac{\partial f}{\partial r} = 0 \quad \text{at} \quad r = R. \tag{6}$$

Boundary condition (5) corresponds to entrainment of the particles with radius R_0 from the apparatus, and condition (6) means that the probability flux through the side wall of the chamber is equal to zero, i.e., that the side wall is impermeable. The boundary condition

$$f(r, 0) = f_0(r), \tag{7}$$

where $f_0(r)$ is a known function satisfying the normalization condition

$$\int_{R_0}^{R} f_0(r) = 1. \tag{8}$$

When the particles are fed through a narrow annulus located at distance r_0 from the chanber axis,

$$f_0(r) = \delta(r - r_0). \tag{9}$$

In the case of a uniform particle distribution at the inlet

$$f_0(r) = \frac{1}{R - R_0}. \tag{10}$$

The radial gas flow is approximated by the relation $V_r(r) = -A/r^m$, where $m = 1$ is assumed for simplicity. The tangential velocity of the particles can be determined from the equations of motion.

First, we consider the case where there is no relative circular motion of phases and the density of particles is constant, i.e., mass transfer in the chamber is neglected. Then, with the notation

$$\omega = \frac{W_\varphi(r)}{r}, \quad x = \frac{r}{R}, \quad P = \frac{A}{BR^2}, \quad \alpha = \frac{BR^2}{C},$$

$$B = \frac{d_p^2(\rho_p - \rho_g)\,\omega^2}{18\mu}$$

Eq. (2) and boundary conditions (5) and (6) are transformed to the form

$$\frac{1}{B}\frac{\partial f}{\partial \tau} = -\frac{\partial}{\partial x}\left[\left(x - \frac{P}{x}\right)f\right] + \frac{1}{2\alpha}\frac{\partial^2 f}{\partial x^2}, \tag{11}$$

$$f = 0 \quad \text{at} \quad x = x_0, \tag{12}$$

$$\left(x - \frac{P}{x}\right)f - \frac{1}{2\alpha}\frac{\partial f}{\partial x} = 0 \quad \text{at} \quad x = 1, \tag{13}$$

$$f(0, x) = \delta(x - x_0). \tag{14}$$

The solution of Eq. (11) will be found by the method of separation of variables, assuming

$$f(x, \tau) = \varphi(\tau)\,\psi(x). \tag{15}$$

Substitution of Eq. (15) into Eq. (11) gives

$$\varphi(\tau) = \exp(-\lambda B\tau), \tag{16}$$

$$\frac{1}{2\alpha}\frac{d^2\psi}{dx^2} - \frac{d}{dx}\left[\left(x - \frac{P}{x}\right)\psi\right] + \lambda\psi = 0, \tag{17}$$

$$\psi = 0 \quad \text{at} \quad x = x_0, \tag{18}$$

$$\left(x - \frac{P}{x}\right)\psi - \frac{1}{2\alpha}\frac{d\psi}{dx} = 0 \quad \text{at} \quad x = 1. \tag{19}$$

Following [5], we introduce the new variables will be introduced

$$z = x^2\alpha, \quad W = z^{-\nu}e^{-z/2}\psi, \quad \nu = -\frac{\alpha P}{2} - \frac{1}{4}.$$

After some transformations we obtain

$$\frac{dW^2}{dz^2} + \left(\frac{\lambda/2 - 1/2 - \nu}{z} - \frac{1/4 - \nu^2}{z^2} - \frac{1}{4}\right)W = 0. \tag{20}$$

It is known [9] that Eq. (20) has two linearly independent solutions:

$$\Phi^{\mathrm{I}}(z) = z^{\nu+1/2} e^{-z/2} \Phi(2\nu - \frac{\lambda}{2} + 1, \ 2\nu + 1; \ z),$$

$$\Phi^{\mathrm{II}}(z) = z^{-\nu+1/2} e^{-z/2} \Phi(-\frac{\lambda}{2} + 1, \ -2\nu + 1; \ z).$$

The eigenfunctions satisfying condition (18) are

$$\psi_i = x^{-2\alpha P} \Phi_i^{\mathrm{II}}(x_0) \Phi_i^{\mathrm{I}}(x) - a_1 x \Phi_i^{\mathrm{I}}(x_0) \Phi_i^{\mathrm{II}}(x), \tag{21}$$

where $a_1 = x_0^{-2\alpha P-1}$.

The degenerate hypergeometric functions are defined by the expressions [9]

$$\Phi^{\mathrm{I}}(x) = 1 + \sum_{n=1}^{\infty} Y_n^{\mathrm{I}}(\lambda) x^{2n}, \tag{22}$$

$$\Phi^{\mathrm{II}}(x) = 1 + \sum_{n=1}^{\infty} Y_n^{\mathrm{II}}(\lambda) x^{2n}, \tag{23}$$

where

$$Y_n^{\mathrm{I}}(\lambda) = \frac{\prod\limits_{i=0}^{n-1} (i - \alpha P - \frac{\lambda}{2} + \frac{1}{2})}{\prod\limits_{i=0}^{n-1} (i - \alpha P + \frac{1}{2})} \frac{\alpha^n}{n!};$$

$$Y_n^{\mathrm{II}}(\lambda) = \frac{\prod\limits_{i=0}^{n-1} (i + 1 - \frac{\lambda}{2})}{\prod\limits_{i=0}^{n-1} (i + \alpha P + \frac{3}{2})} \frac{\alpha^n}{n!}.$$

With Eq. (21), we have from condition (19)

$$\Phi^{\mathrm{II}}(x_0) [\Phi^{\mathrm{I}}(1) - F^{\mathrm{I}}(\lambda)] - \Phi^{\mathrm{I}}(x_0) [a_2 \Phi^{\mathrm{II}}(1) - F^{\mathrm{II}}(\lambda)] = 0, \tag{24}$$

where

$$F^{\mathrm{I}}(\lambda) = \frac{1}{\alpha} \sum_{n=1}^{\infty} Y_n^{\mathrm{I}}(\lambda) n,$$

$$F^{\mathrm{II}}(\lambda) = \frac{a_1}{\alpha} \sum_{n=1}^{\infty} Y_n^{\mathrm{II}}(\lambda) n, \quad a_2 = a_1 \left(1 - P - \frac{1}{2\alpha}\right).$$

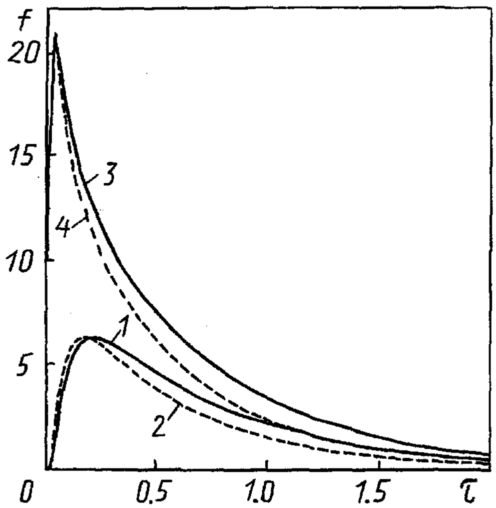
The eigenvalues λ_i are a solution of Eq. (24) and the eigenfunction ψ_i correspond to each of the eigenvalues. Then, the solution of Eq. (11) has the form

$$f(x, \tau) = \sum_{i=1}^{\infty} C_i \psi_i \exp(-\lambda_i B\tau). \tag{25}$$

The constants C_i will be determined from initial condition (14). The functions ψ_i are orthogonal in the weight $x^{2\alpha P} e^{-\alpha x^2}$ [10]. Consequently,

$$C_i = \frac{1}{N_i} \int_{x_0}^{1} f_0(x) x^{2\alpha P} \exp(-\alpha x^2) \psi_i \, dx, \tag{26}$$

$$N_i = \int_{x_0}^{1} \psi_i^2 x^{2\alpha P} \exp(-\alpha x^2) \, dx.$$



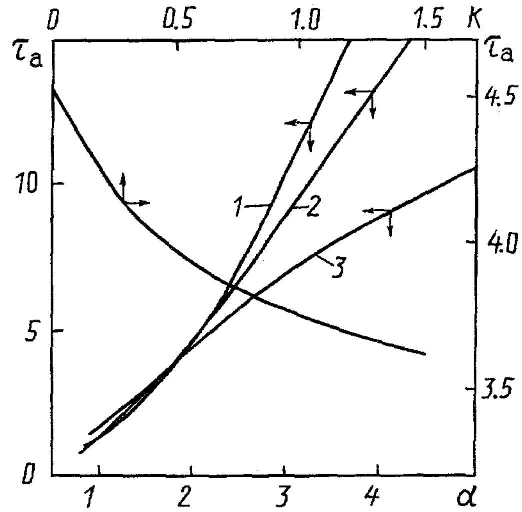


Fig. 2. Differential curves of particle distribution with respect to residence time at different r/R and $\omega = 150\ \text{sec}^{-1}$: $A = 0.04$; $P = 1.01$; $K = 0$; 1) $r/R = 0.4$; $\alpha = 1.98$; 2) 0.4 and 1.58; 3) 0.8 and 1.98; 4) 0.8 and 1.58. τ, sec.

Fig. 3. Plot of the average particle residence time in the chamber versus the process parameters: 1) $P = 1.01$; 2) 1.16; 3) 1.46. τ_a, sec.

Eventually, formulas (21)-(26) determine the unknown function of the density distribution $f(x, \tau)$.

If the particle density depends on time, for example, in the drying of particles and also in the case of relative tangential motion of the phases, the solution of the problem is more complicated. Therefore, Eq. (2) was solved by the finite-difference method. In the numerical experiment the process parameters α, P, and K were varied in a sufficiently wide range to reveal the character of evolution of the distribution function. The average residence times of particles in the given cross section and in the whole apparatus were determined from the relations

$$\bar{\tau} = \int_0^\infty f(x, \tau)\, \tau\, d\tau, \quad \tau_a = \frac{1}{1 - x_0} \int_{x_0}^1 \bar{\tau}\, dx.$$

Calculations were carried out with the following constant parameters: $\rho_p = 900$ kg/m^3; $d_p = 4 \cdot 10^{-6}$ m; $\rho_g = 1.2$ kg/m^3; $R_0 = 0.075$ m; $R = 0.2$ m; $\mu = 1.82 \cdot 10^{-5}$ Pa·sec; $u_{in} = 0.26$ kg/kg; $u_{eq} = 0.02$ kg/kg. In Fig. 2 one can see the differential curves of particle distribution with respect to residence time at various process parameters. The parameter α characterizes the ratio of centrifugal and mixing forces caused by random interactions, and the parameter $P = A(BR^2)$ determines the ratio of intensities of the radial resistance forces and centrifugal forces. As one would expect, as the intensity of the random forces increases and α decreases, the distribution curves appear more and more smeared with decrease in the average particle residence time in the apparatus (Fig. 3). As P decreases, the curves are shifted toward longer times and, consequently, toward higher average τ_a. Figure 4 shows that the curve of the particle residence time versus the chamber radius is parabolic. In this case, the residence time in the wall region is much longer than that in the central area. This fact is consistent with a higher concentration of disperse particles in the wall region, i.e., with formation of a rotating annular layer of particles, and agrees with experimental observations. As α decreases and the drying rate increases, the particle residence time in the wall region is reduced. The reduction of the particle residence time in the chamber with increase in the drying rate, for example, as a result of increase in the gas temperature, is associated with decrease in the particle density, which results in weaker centrifugal forces. Increase in the diameter of the inlet orifice also results in decrease in the average time [7].

Thus, analysis has shown a substantial effect of the process parameters considered, including mass transfer rate, on the evolution of the distribution functions and their characteristics. Since they determine the particle distribution with respect to moisture content and temperature, i.e., determine the quality of the product, the character of the effect of these parameters should be taken into consideration in the design of vortex apparatus.

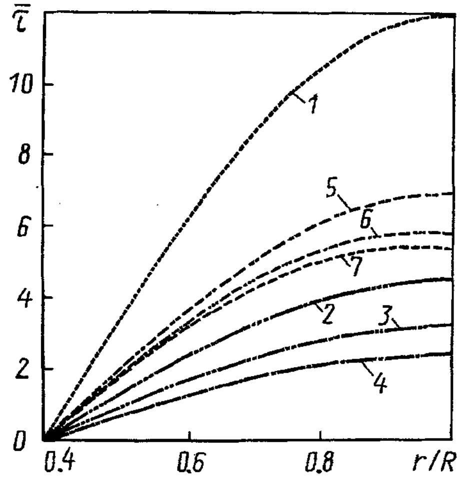


Fig. 4. The distribution of the particle residence time over the radius of the chamber: 1) $\alpha = 2.63$; 2) 1.58; 3) 1.31; 4) 1.13 at $P = 1.01$; $\omega = 150$ sec^{-1}; $K = 0$; $A = 0.04$; 5) $K = 0$; $\alpha = 1.98$; 6) $K = 0.5$; 7) 1.0. $\bar{\tau}$, sec.

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NOTATION

A, constant; C', parameter characterizing the intensity of random forces; d_p, particle diameter, m; K, drying rate coefficient; r, radial coordinate of the particle, m; R_0, radius of the outlet orifice, m; R, radius of the chamber, m; u, u_{in}, u_{eq}, instantaneous, initial, and equilibrium moisture contents of the particle, kg/kg; V_r, radial gas velocity, m/sec; W_φ, tangential velocity of the particle, m/sec; $x = r/R$, dimensionless variable; μ, dynamic viscosity, Pa·sec; ρ_p, ρ_g, density of the particles and gas, kg/m^3; τ, time, sec; ω, angular velocity of gas suspension, sec^{-1}.

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